**AP CALCULUS BC COURSE SYLLABUS**

Textbook: Calculus for AP , 2nd Edition by R. Larson & P. Battaglia

# Marking Period 1

## **Unit 1: Limits and Continuity** (Test 1.2 - 1.6)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 1.2 | Finding Limits Graphically and Numerically | F-IF.C | Pg: 72 - 75 |
| 1.3 | Evaluating Limits Analytically | A.SSE.B | Pg: 84 - 87 |
| 1.4 | Continuity & One-Sided Limits | F.IF.C | Pg. 96-99 |
| 1.5 | Infinite Limits | F.IF.C | Pg. 105-107 |
| 1.6 | Limits at Infinity | F.IF.C | Pg. 115-117 |

## **Unit 2: Differentiation** (Test 2.1 - 2.7 & 7.7)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 2.1 | The Derivative & Tangent Line Problem | A.SSE.B | Pg. 132-134 |
| 2.2 | Basic Differentiation Rules & Rates of Change | A.SSE<br>A.APR | Pg. 144-145 |
| 2.3 | Product & Quotient Rules and Higher Order Derivatives | A.SSE<br>A.APR | Pg. 155-158 |
| 2.4 | Chain Rule | A.SSE<br>A.APR | Pg. 169-173 |
| 2.5 | Implicit Differentiation | A.SSE<br>F.IF | Pg. 180-182 |
| 2.6 | Derivatives of Inverse Functions | F.BF.B<br>F.IF | Pg. 187-189 |
| 2.7 | Related Rates | F.BF<br>F.LE | Pg. 195-198 |
| 7.7 | Indeterminate Forms & L'Hopital's Rule | A.SSE | Pg. 513-516 |

**Unit 3: Applications of Derivatives** (Test 3.1 - 3.7)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 3.1 | Extrema on an Interval | A.REI | Pg. 217-219 |
| 3.2 | Rolle's & Mean Value Theorem | A.REI | Pg. 224-226 |
| 3.3 | Increasing & Decreasing Functions and the 1st Derivative Test | A.CED.A | Pg. 233-236 |
| 3.4 | Concavity & the 2nd Derivative Test | A.CED.A | Pg. 242-244 |
| 3.5 | Summary of Curve Sketching | A.REI.D | Pg. 253-256 |
| 3.6 | Optimization | A.REI.C | Pg. 262-266 |
| 3.7 | Linear Approximation & Differentials | A.SSE<br>F-BF | Pg. 272-273 |

## Marking Period 2

**Unit 4: Integration** (Test 4.1 - 4.3, Test 4.6-4.8, 7.2-7.5)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 4.1 | Antiderivatives & Indefinite Integration | A.SSE | Pg. 287-289 |
| 4.2 | Area Under a Curve | G.MG.A3 | Pg. 299-301 |
| 4.3 | Riemann Sums & Definite Integrals | A.SSE.B<br>G.MG.A3 | Pg. 312-315 |
| 4.6 | Integration by Substitution | A.CED.A | Pg. 343-346 |
| 4.7 | Natural Logarithm Function: Integration | F.LE.A | Pg. 353-355 |
| 4.8 | Inverse Trigonometric Function: Integration | F.TF.B.7 | Pg. 361-363 |
| 7.2 | Integration by Parts | A.REI.A<br>A.REI.C | Pg. 469-472 |
| 7.3 | Trigonometric Integrals | F.TF.C | Pg. 479-481 |
| 7.4 | Trigonometric Substitution | F.TF.C | Pg. 488-490 |
| 7.5 | Partial Fractions (Linear Only) | A.APR.D.6 | Pg. 498-499 |

**Unit 5: Applications of Integration** (Test 6.1, 6.2, 6.4)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 4.4 | Fundamental Theorem of Calculus | A.SSE.B<br>G.MG.A3 | Pg. 326-328 |
| 6.1 | Area of a Region Between Two Curves | A.SSE.B<br>G.MG.A3 | Pg. 416-419 |
| 6.2 | Volume: Washer & Disc Method | G.GMD.A3 | Pg. 427-430 |
| 6.4 | Arc Length & Surfaces of Revolution | G.GPE.B.7 | Pg. 446-449 |

**Unit 6: Differential Equations** (Test 5.1 - 5.4)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 5.1 | Slope Fields and Euler's Method | S.ID.C | Pg. 375-378 |
| 5.2 | Growth & Decay | F.IF.C.8b | Pg. 384-386 |
| 5.3 | Separation of Variables | A.APR.D | Pg. 393-396 |
| 5.4 | Logistic Equation | A.APR.D | Pg. 402-403 |

## Marking Period 3

**Unit 7: Parametric Equations, Polar Coordinates, & Vectors** (Test 9.2,9.4 & 9.6, Test 9.3,9.5,9.7,9.8)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 9.2 | Plane Curves & Parametric Equations | A.REI.D | Pg: 652 - 654 |
| 9.4 | Polar Coordinates & Polar Graphs | F.IF.C | Pg. 670 - 672 |
| 9.6 | Vectors in the Plane | N.VM.B | Pg. 686 - 688 |
| 9.3 | Parametric Equations & Calculus | A.SSE.B | Pg. 659 - 662 |
| 9.5 | Area and Arc Length in Polar Coordinates | F.IF.C | Pg. 678 - 680 |
| 9.7 | Vectored Value Functions | N.VM.A | Pg. 695 - 697 |
| 9.8 | Velocity & Acceleration | N.VM.A.3 | Pg. 702 - 703 |

**Unit 8: Infinite Series** (Test 8.1 - 8.6 & 8.7 - 8.10)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 8.1 | Sequences | F.IF.A.3 | Pg. 542 - 544 |
| 8.2 | Series and Convergence | A.SSE.B4<br>F.IF.A.3 | Pg. 552 - 554 |
| 8.3 | Integral Test and P-Series | A.REI.A | Pg. 559 - 561 |
| 8.4 | Comparison of Series | A.REI.D | Pg. 566 - 568 |
| 8.5 | Alternating Series | A.REI.D | Pg. 575 - 576 |
| 8.6 | The Ratio & Root Tests | A.REI.D | Pg. 583 - 585 |
| 8.7 | Taylor Polynomials & Approximations | A.APR.C | Pg. 594 - 596 |
| 8.8 | Power Series | A.REI.A<br>A.APR.C | Pg. 604 - 606 |
| 8.9 | Representation of Functions by Power Series | A.REI.A<br>A.APR.C | Pg. 612 - 613 |
| 8.10 | Taylor & MacLaurin Series | A.REI.A<br>A.APR.C | Pg. 623 - 625 |

## Marking Period 4

**Unit 9: AP Test Review** (Test Mock AP Test)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| Supplement | Units 1 - 8 | | AP Central FRQ & MCQ |

**Unit 10: Additional Topics** (Test4.5 & 6.3)

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| 4.5 | The Net Change Theorem | A.SSE.B | Pg. 333 - 335 |
| 6.3 | Shell Method | G.GMD.A3 | Pg. 436 - 439 |

## Unit 11: Project

| Section | Title | NJSLS | Problems |
|---|---|---|---|
| Supplement | Marginal Analysis Project | A.CED.A | Activity Sheet |

**Course Description:**
AP Calculus BC is representative of a college-level calculus course. Students cultivate their understanding of differential and integral calculus through engaging with real-world problems represented graphically, numerically, analytically, and verbally and using definitions and theorems to build arguments and justify conclusions as they explore concepts like change, limits, and the analysis of functions.

**Course Expectations and Skills**
• Students are required to have proficiency in all prerequisite topics for Calculus. Those who do not demonstrate proficiency will be required to seek additional help after school to close their achievement gap in order to be successful in this course.
• Students are to bring their Chrome Book charged and ready to use for the lesson.
• Students are encouraged to have a graphing calculator.
• Students are required to participate in both small and large group discussions and activities, as directed.

**Resources**
Text Book: Calculus for AP , 2nd Edition by R. Larson & P. Battaglia

**Additional Resources:**
• https://apcentral.collegeboard.org/
• https://sso.cengage.com/cb/
• WebAssign
• CalcChat.com
• CalcView.com
• Desmos.com

| Grading Breakdown | |
|---|---|
| Major Summative (MAJ) | 60% |
| Minor Formative (MIN) | 30% |
| Homework (HW) | 10% |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC

## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Limits and Continuity** | |
|---|---|
| In this unit students develop an understanding of limits as the foundational building blocks for both derivatives and integration. It is essential for discovering and developing important ideas, definitions, formulas and theorems in calculus. Students will solve limit problems graphically, algebraically, and conceptually. They will generate and work with tables, sketch and analyze various graphs, and apply numerous algebraic techniques to find limits of indeterminate forms. Students must have a solid, intuitive understanding of limits and be able to compute various limits, such as, one-sided limits, limits at infinity, infinite limits, and trigonometric limits. In addition, they will communicate both orally and in written form effectively what their answers mean in the context of the problems they are given. Finally, students will understand how limits are used to determine continuity, which is a fundamental property of functions, and apply the Intermediate Value Theorem. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What is a limit and how can you determine the limit of a function as x approaches c?<br>2. What algebraic techniques can you use to evaluate a limit?<br>3. What is continuity and how does it apply to the Intermediate Value Theorem?<br>4. What is an infinite limit? | 1. Reasoning with definitions, theorems and properties can be used to justify claims about limits.<br>2. Reasoning with definitions, theorems and properties can be used to justify claims about continuity. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Function<br>2. Continuity<br>3. Infinite<br>4. Asymptote<br>5. Removable Discontinuity | 1. Limit |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Estimate a limit using a numerical or graphical approach<br>2. Evaluate a limit using properties of limits<br>3. Determine Continuity at a point and continuity on an open interval<br>4. Determine one-sided limits and continuity on a closed interval<br>5. Determine Infinite limits from the left and the right<br>6. Determine limits at infinity | 1. HSF-IF.C<br>2. HSA.SSE.B<br>3. HSF-IF.C<br>4. HSF-IF.C<br>5. HSF-IF.C<br>6. HSF-IF.C | 1. 2B<br>2. 1C, 1E, 2C, 3C<br>3. 1E, 3B, 3C, 3E<br>4. 3C<br>5. 1E, 3D<br>6. 2D |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 1.2* ***Page**: 74*<br>***Description:*** Cost analysis functions for paddle board company | Estimate a limit using numerical or graphical approach | CLKS 9.1 |
| ***Section**: 1.3* ***Page**: 86*<br>***Description:*** Comparing velocity and position functions to make predictions | Evaluate a limit using properties of limits | SCI.HS-PS2 |
| ***Section**: 1.4* ***Page**: 91*<br>***Description:*** Using Charles's Law and absolute value to determine a lower limit | Determine one-sided limits and continuity on a closed interval. | SCI.HS-ESS3-5 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| **Students will write:** |
| --- |
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| What the domain of a function is if given algebraically.<br>What possible errors can be made by determining the domain of a function solely by analyzing a function's graph.<br>The importance of examining a function analytically as well as graphically.<br>A brief description of the meaning to the notation $\lim_{x \to c} f(x)$<br>Compare $f(x) = x$, $g(x) = sinx$, and $h(x) = \frac{sinx}{x}$ graphically and write why $\lim_{x \to 0} h(x) = 1$.<br>Compare $f(x) = x$, $g(x) = x$, and $h(x) = \frac{x}{x}$ graphically and write why $\lim_{x \to 0} h(x) = 0$.<br>About the importance of examining a function analytically as well as graphically when determining continuity.<br>Descriptions about how functions differ.<br>Meanings of different types of discontinuity and explain why.<br>Explanations as to why the intermediate value theorem applies on a given closed interval. |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**<u>How</u> will students uncover content and build skills?**

| 1.2 - Finding Limits Graphically and Numerically | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Represent limits analytically using correct notation<br>• Interpret limits expressed ns in analytic notation<br>• Estimate limits of functions | Determine the Vertical Asymptotes for the given functions | Larson Text Section: 1.2 Pg: 72 - 75 |

| 1.3 - Evaluating Limits Analytically | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Determine the limits of functions using limit theorems<br>• Determine the limits of functions using equivalent expressions for the function or the squeeze theorem. | Interpret the given limit and estimate its value | Larson Text Section: 1.3 Pg: 84 - 87 |

| 1.4 - Continuity and One-Sided Limits | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Justify conclusions about continuity at a point using the definition<br>• Determine intervals over which a function is continuous<br>• Determine values of x or solve for parameters that make discontinuous functions continuous, if possible<br>• Explain the behavior of a function on an interval using the IVT | Determine the value of the given limit and verify graphically using calculator | Larson Text Section: 1.4 Pg: 96 - 99 |

| 1.5 - Infinite Limits | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Interpret the behavior of functions using limits involving infinity | Define asymptote. What is a function's value at an asymptote? Explain. | Larson Text Section: 1.5 Pg: 105 - 107 |

| 1.6 - Limits at Infinity | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Interpret the behavior of functions using limits involving infinity | In your own words, describe the meaning of an infinite limit. Is infinity a real number? | Larson Text Section: 1.6 Pg: 115 - 117 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Projects<br>● End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>● Projects<br>● Practice AP Exam Questions<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● ELL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Career Readiness, Life Literacies, and Key Skills<br>● LGBT and Disabilities Law<br>● Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| Unit Title: Differentiation | |
|---|---|
| In this unit students will get into the heart of calculus. Derivatives are a major portion of the course, so they will spend a significant amount of time in this unit. Derivatives are used to describe the rate of change of one variable with respect to another to understand change in a variety of contexts. At first students build the derivative using the concept of limits and use it to progamily compute the instantaneous rate of change of a function. Students should be able to use different definitions of derivatives, estimate derivatives from tables and graphs, and apply various derivative rules and properties. As they progress through this unity they will spend some time on the relationship between position, velocity, and acceleration on problems involving projectile motion and rectilinear motion. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What is a derivative and what is the relationship of continuity?<br>2. How do you find the derivatives of basic algebraic functions, trigonometric functions, and exponential functions?<br>3. How do you find the derivatives of functions involving products and quotients?<br>4. How do you find the derivatives of composite functions, natural logarithmic functions, and exponential functions with bases other than e?<br>5. How do you find the derivative of implicitly defined functions?<br>6. How do you find the derivatives of inverse functions, including inverse trigonometric functions?<br>7. What is a related rate and how do you find it? | 1. Determine average rates of change using difference of quotients<br>2. Represent the derivative as a limit of a difference quotient<br>3. Calculate and interpret equations of tangent lines to a curve<br>4. Calculate derivatives and higher order derivative for familiar functions and composite functions<br>5. Calculate and interpret derivatives of implicitly defined functions<br>6. Calculate and interpret related rates in applied contexts<br>7. Determine limits of functions utilizing L'Hopital's Rule. |

| | |
|---|---|
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Limit<br>2. Continuity<br>3. Tangent<br>4. Secant<br>5. Linear Equation<br>6. Function | 1. Derivative<br>2. Difference Quotient<br>3. Related Rate |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| **Unit Learning Targets/Goals/Outcomes** | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Find the slope of the tangent line to a curve at a point<br>2. Use derivatives to find Rates of change<br>3. Find derivatives using the Product and Quotient rules<br>4. Find derivatives using the Chain Rule<br>5. Use implicit differentiation to find the derivative of an implicit function<br>6. Find the derivative of an inverse function<br>7. Find and utilize a a related rate to solve real-life problems<br>8. Apply L'Hopital's Rule to evaluate a limit | 1. HSA.SSE.B<br>2. HSA.SSE<br>3. HSA.APR<br>4. HSA.APR<br>5. HSF.IF<br>6. HSF.IF<br>7. HSF.LE<br>8. HSA.SSE | 1. 1.D, 2.B, 4.C<br>2. 1.E, 3.E<br>3. 2.8<br>4. 1.C<br>5. 1.E<br>6. 1.3, 3.G<br>7. 1.C, 1.E, 3.F<br>8. 1.C, 3.D |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 2.1* ***Page**: 124*<br>***Description:*** Tangent Line Problem - Optics and Refraction | Find the slope of the tangent line to a curve at a point | HS-PS4-4 |
| ***Section**: 2.3* ***Page**: 157*<br>***Description:*** Particle Motion | Find derivatives using the Product and Quotient Rules | HS-PS2-1 |
| ***Section**: 7.7* ***Page**: 515*<br>***Description:*** Electric Circuit Diagram | Apply L'Hopital's Rule to evaluate a limit | HS-PS1-3 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| Descriptions of the geometric significance of differentiability of two functions. The meaning of $f'(1.4)$ given $N=f(p)$ is the function that yields the number of gallons of gasoline sold by a station at a price of dollars/gal<br>The criteria used in selecting a graph as $f$ and $f'$ when given two graphs.<br>Justifications as to why or why not a function has a tangent line at a given point.<br>The criteria for a function to have a horizontal tangent present.<br>Connections to differentiability and continuity.<br>Appropriate units with all answers that model real world situations – On the AP exam students may not earn a point on a free-response question when units are not included with the solution.<br>Explanations how the velocity of an object can be determined given a graph of its position.<br>Describe the difference between the explicit form of a function and an implicit equation.<br>The difference between a negative rate of change and a positive rate of change.<br>Explain if x changes at a constant rate, does y change at a constant rate and if so is it the same as x.<br>Explain and justify their approach for solving related-rate problems. |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 2.1 - The Derivative and The Tangent Line Problem | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find the slope of the tangent line to a curve at a point<br>• Use the limit definition to find the derivative of a function<br>• Understand the relationship between differentiability & continuity<br>• Find the derivative of a function given by a table or a graph | Find the slope of the given secant line. Can you find slope with only 1 point? | Larson Text Section: 2.1 Pg: 132-134 |

| 2.2 - Basic Differentiation Rules and Rates of Change | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find the derivative of a function using various Differentiation techniques<br>• Find the derivative of trigonometric functions<br>• Find the derivative of exponential functions<br>• Use derivatives to find rates of change | Solve the following using first principles. | Larson Text Section: 2.2 Pg: 144-147 |

| 2.3 - Product & Quotient, Trigonometric, and Higher-Oder Derivatives | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find the derivative of a function using the Product Rule<br>• Find the derivative of a function using the QUotient Rule<br>• Find the derivative of a trigonometric function<br>• Find a higher-order derivative of a function | Find the equation of the tangent line | Larson Text Section: 2.3 Pg: 155-158 |

| 2.4 - The Chain Rule and Exponential Functions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find the derivative of a composite function using the Chain Rule<br>• Find the derivative of a function using the General Power Rule<br>• Simplify the derivative of a function using algebra<br>• Find the derivative of a transcendental function using the Chain Rule<br>• Find the derivative of a function involving the natural logarithmic function<br>• Define and differentiate exponential functions that have bases other than e. | Determine f'(x) and dy/dx of the given functions | Larson Text Section: 2.4 Pg: 169-173 |

| 2.5 - Implicit Differentiation and Logarithmic Differentiation | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Distinguish between functions written in implicit form and explicit form<br>• Use implicit differentiation to find the derivative of a function<br>• Find derivatives of functions using logarithmic differentiation | Determine f'(x) of the given functions | Larson Text Section: 2.5 Pg: 180-182 |

| 2.6 - Derivatives of Inverse Functions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find the derivative of an inverse function<br>• Differentiate an inverse trigonometric function | Solve for dy/dx. (use implicit differentiation) | Larson Text Section: 2.6 Pg: 187-189 |

| 2.7 - Related Rates | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Find a Related Rate<br>• Use related rates to solve real-life problems | Determine if A, B, C, or D is the solution to the given problems (MC AP Practice) | Larson Text Section: 2.7 Pg: 195-198 |

| 7.7 - Indeterminate Forms and L'Hopital's Rule | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Recognize Limits that produce indeterminate forms<br>• Apply L'Hopital's Rule to evaluate a limit | Solve the given limit using an algebraic approach | Larson Text Section: 7.7 Pg: 513-516 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
<tr>
<td>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<ul><li>Diagnostic Pre-Test</li><li>Chapter Tests</li><li>Projects</li><li>End-Of –Course Assessment</li></ul></td>
<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.</td>
<td>The following assessments require students to utilize various strands of mathematics.<ul><li>Projects</li><li>Practice AP Exam Questions</li><li>Homework</li><li>Classwork</li></ul></td>
</tr>
<tr>
<td colspan="3">List of Accommodations and Modifications<ul><li>Special Education</li><li>504 Students</li><li>At Risk Students</li><li>ELL</li><li>Gifted and Talented</li></ul></td>
</tr>
</table>

| State Mandates and Resources |
|---|
| • New Jersey Student Learning Standards<br>• Career Readiness, Life Literacies, and Key Skills<br>• LGBT and Disabilities Law<br>• Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

## Practice 1

### *Implementing Mathematical Processes* 1

Determine expressions and values using mathematical procedures and rules.

## Practice 2

### *Connecting Representations* 2

Translate mathematical information from a single representation or across multiple representations.

## Practice 3

### *Justification* 3

Justify reasoning and solutions.

## Practice 4

### *Communication and Notation* 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

## SKILLS

**1.A** Identify the question to be answered or problem to be solved *(not assessed)*.

**1.B** Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

**1.C** Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

**1.D** Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

**1.E** Apply appropriate mathematical rules or procedures, with and without technology.

**1.F** Explain how an approximated value relates to the actual value.

**2.A** Identify common underlying structures in problems involving different contextual situations.

**2.B** Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

**2.C** Identify a re-expression of mathematical information presented in a given representation.

**2.D** Identify how mathematical characteristics or properties of functions are related in different representations.

**2.E** Describe the relationships among different representations of functions and their derivatives.

**3.A** Apply technology to develop claims and conjectures *(not assessed)*.

**3.B** Identify an appropriate mathematical definition, theorem, or test to apply.

**3.C** Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

**3.D** Apply an appropriate mathematical definition, theorem, or test.

**3.E** Provide reasons or rationales for solutions and conclusions.

**3.F** Explain the meaning of mathematical solutions in context.

**3.G** Confirm that solutions are accurate and appropriate.

**4.A** Use precise mathematical language.

**4.B** Use appropriate units of measure.

**4.C** Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$).

**4.D** Use appropriate graphing techniques.

**4.E** Apply appropriate rounding procedures.

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| Unit Title: Applications of Differentiation | |
|---|---|
| In this unit, students will go on to understand and apply the Mean Value Theorem and will have the opportunity to see how the average rate of change can be used to justify instantaneous speed. They will also spend a significant amount of time developing a comprehensive analysis of functions using not only their graphs but their derivatives as well. Students should be familiar with a variety of real-world applications, including related rates, optimization, linear approximation, and growth and decay models. This is most likely the first time students will be asked to think deeply on a conceptual level, so they may struggle to make connections at first. Students will also learn how far to simplify solutions and provide meaningful simplifications to clarify solutions. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What are extrema and how can you find them on open and closed intervals?<br>2. What is the Mean Value Theorem and how is it used?<br>3. How can you determine the intervals on which a function is increasing or decreasing and the location of the function's relative extrema?<br>4. How do you determine the concavity of a function and find its inflection points?<br>5. How do you analyze a function and sketch its graph?<br>6. How do you maximize or minimize quantities?<br>7. How are differentials used to explain the tangent line approximation? | 1. Justify conclusions about functions by applying the Extreme Value Theorem<br>2. Justify conclusions about the behavior of a function based on the behavior of its derivatives<br>3. Determine critical points of implicit relations<br>4. Justify conclusions about the behavior of a function based on the behavior of its derivatives<br>5. Calculate minimum and maximum values in applied contexts or analysis of functions<br>6. Interpret minimum and maximum values calculated in applied contexts<br>7. Approximate a value on a curve using the equation of a tangent line |

| Tier 2 Vocabulary<br>*High-frequency words used throughout the unit* | | Tier 3 Vocabulary<br>*Discipline-specific words used throughout the unit* | |
|---|---|---|---|
| 1. Limit<br>2. Continuity<br>3. Derivative | 4. Interval<br>5. Increase/Decrease | 1. Concavity<br>2. MVT<br>3. Rolle's Theorem | 4. Extrema<br>5. Inflection Point |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Understand the definition of extrema of a function on an open and closed interval | 1. HSA.REI | 1. 3.E |
| 2. Understand both Rolle's and the Mean Value Theorems. | 2. HSA.REI | 2. 3.E |
| 3. Determine intervals on which a function is increasing or decreasing | 3. HSA.CED.A | 3. 2.E, 3.D |
| 4. Determine intervals on which a function is concave up or down | 4. HSA.CED.A | 4. 2.E, 3.D |
| 5. Analyze and sketch the graph of a function | 5. HSA.REI.D | 5. 2.D |
| 6. Solve applied minimum and maximum problems. | 6. HSA.REI.C | 6. 1.E, 3.E, 3.F |
| 7. Understand the concept of a tangent line approximation | 7. HSA.SSE, HSF-BF | 7. 3.E |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 3.2* ***Page**: 223*<br>***Description:*** Finding instantaneous rate of change of cars | Understand both Rolle's and the Mean Value Theorems. | HS-PS2-1 |
| ***Section**: 3.4* ***Page**: 243*<br>***Description:*** Highway design - Engineering | Determine intervals of which a function is concave up or down | HS-PS1-3 |
| ***Section**: 3.6* ***Page**: 262*<br>***Description:*** Farming plots | Solve applied minimum and maximum problems | HS-ESS3-4 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| How derivatives can be used to approximate the zero of a function. The difference between relative and absolute extrema.<br>The graph of a function on a closed interval given extrema locations.<br>Explain why or why not the Mean Value Theorem applies to a function on a closed interval.<br>Explain how the Mean Value Theorem and Rolle's theorem are similar and different.<br>How you can determine the intervals on which a function is increasing or decreasing.<br>How you can determine the location of a function's relative extrema using derivatives.<br>How to determine the concavity of a function using derivatives.<br>How can derivatives be used to sketch the graph of a function precisely?<br>Justify the possibility of a function crossing its horizontal or vertical asymptotes.<br>To explain since the surface area of a bottle does not change when squeezed, how does the volume change? Justify how to maximize or minimize quantities and apply appropriate units. |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 3.1 - Extrema on an Interval | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Understand the definition of extrema of a function on an interval<br>• Understand the definition of relative extrema of a function on an open interval<br>• Find extrema on a closed interval | Find the coordinate of the vertices for the given function | Larson Text Section: 3.1 Pg: 217-219 |

| 3.2 - Rolle's Theorem and the Mean Value Theorem | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Understand and use Rolle's Theorem<br>• Understand and use the Mean Value Theorem | Determine the max or min value of the function and determine if its a local or extrema | Larson Text Section: 3.2 Pg: 224-226 |

| 3.3 - Increasing and Decreasing Functions and the First Derivative Test | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Determine intervals on which a function is increasing or decreasing<br>• Apply the First Derivative Test to find relative extrema of a function | Use the MVT to verify that an extrema point exists on the given interval | Larson Text Section: 3.3 Pg: 233-236 |

| 3.4 - Concavity and the Second Derivative Test | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Determine intervals on which a function is concave upward or concave downward.<br>• Find any points of inflection of the graph of a function<br>• Apply the Second Derivative Test to find relative extrema of a function | Determine if the function is increasing or decreasing on the given interval. Verify by graphical analysis. | Larson Text Section: 3.4 Pg: 242-244 |

| 3.5 - A Summary of Curve Sketching | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Analyze and Sketch the graph of a function | What is concavity? Determine the intervals of concavity for the given graph. | Larson Text Section: 3.5 Pg: 253-256 |

| 3.6 - Optimization Problems | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Solve applied minimum and maximum problems. | Determine the intervals of increase and decrease and concavity given the following family of curve sketches. | Larson Text Section: 3.6 Pg: 262-265 |

| 3.7 - Linear Approximation and Differentials | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Understand the concept of tangent line approximation<br>• Compare the value of the differential, dy, with the actual change in y, delta y<br>• Estimate a propagated error using a differential<br>• Find the differential of a function using differentiation formulas. | Various optimization problems. | Larson Text Section: 3.7 Pg: 272-275 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>• Diagnostic Pre-Test<br>• Chapter Tests<br>• Projects<br>• End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>• Projects<br>• Practice AP Exam Questions<br>• Homework<br>• Classwork |

**List of Accommodations and Modifications**

- Special Education
- 504 Students
- At Risk Students
- ELL
- Gifted and Talented

## State Mandates and Resources

- New Jersey Student Learning Standards
- Career Readiness, Life Literacies, and Key Skills
- LGBT and Disabilities Law
- Asian and Pacific Islander

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

### Unit 4: Integration

In this unit, students will establish the relationship between differentiation and integration. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines the accumulation of change over an interval, just as differentiation determines instantaneous rate of change at a point. Students will understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes).

| Essential Questions | Enduring Understandings |
|---|---|
| 1. What are antiderivatives and how are they used?<br>2. How can you approximate the area of a plane region?<br>3. How are Reimann sums similar to the Trapezoidal Rule and how are they different?<br>4. How do you integrate composite functions?<br>5. How do you integrate rational functions and trigonometric functions other than sine and cosine?<br>6. What is integration by parts and when is it used?<br>7. How do you evaluate trigonometric integrals involving powers?<br>8. How can trigonometric substitution be used to solve an integral?<br>9. How do you integrate a complex rational function? | 1. Definite integrals allow us to solve problems involving the accumulation of change over an interval.<br>2. Definite integrals can be approximated using geometric and numerical methods.<br>3. Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.<br>4. The use of limits allows us to show that the areas of unbounded regions may be finite. |

| Tier 2 Vocabulary<br>*High-frequency words used throughout the unit* | | | Tier 3 Vocabulary<br>*Discipline-specific words used throughout the unit* | | |
|---|---|---|---|---|---|
| Derivative | Limits/Continuity | Area | Antiderivatives<br>Riemann Sum | Integration<br>Sigma Notation | Partial Fractions<br>Integration by Parts |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Write the general solution of a differential equation and use indefinite integral notation for antiderivatives. | 1. HSA.SSE | 1. 4B |
| 2. Approximate the area of a plane region and find the area of a plane region using limits. | 2. HSG.MG.A3 | 2. 1F |
| 3. Understand the definition of a Riemann Sum. | 3. HSA.SSE.B, HSG.MG.A3 | 3. 1F |
| 4. Evaluate a definite integral using limits, geometric formulas and properties. | 4. HSA.SSE.B, HSG.MG.A3 | 4. 1F |
| 5. Use pattern recognition, a change of variables and the General Power Rule to find indefinite integrals. | 5. HSA.CED.A | 5. 1E, 4C |
| 6. Use the Log Rule to integrate rational functions and integrate trigonometric functions. | 6. HSF.LE.A | 6. 4C |
| 7. Integrate functions whose antiderivatives involve inverse trigonometric functions. | 7. HSF.TF.B.7 | 7. 1E |
| 8. Find the antiderivative using integration by parts. | 8. HSA.REI.A, .C | 8. 1E |
| 9. Solve trigonometric integrals involving powers of sine, cosine, secant and tangent. | 9. HSF.TF.C | 9. 1E |
| 10. Use trigonometric substitution to find an integral. | 10. HSF.TF.C | 10. 1E |
| 11. Understand and use the concept of partial fraction decomposition to integrate rational functions. | 11. HSA.APR.D6 | 11. 1E |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| **Section:** 4.1 **Page:** 288<br>**Description:** Time-distance of Grand Canyon | Write the general solution of a differential equation | SCI.HS.ESS3 |
| **Section:** 4.7 **Page:** 355<br>**Description:** Heat Transfers (Newton's Cooling Law) | Use the Log Rule to integrate rational functions and integrate trigonometric functions. | SCI.HS.ESS1 |
| **Section:** 7.5 **Page:** 499<br>**Description:** Chemical Reaction | Understand and use the concept of partial fraction decomposition to integrate rational functions. | SCI.HS.PS1 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| - Explain the difference, as you understand it so far, between definite integral and indefinite integral.<br>- The difference between a differential and a derivative.<br>- The difference between Mean Value Theorem and the Intermediate Value Theorem.<br>- What are the different types of Riemann sums?<br>- What evidence can you think of to show that Riemann sums really do get to the value of a definite integral found by the fundamental theorem as n approaches infinity? |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE**

**How will students uncover content and build skills?**

| 4.1 Antiderivatives and Indefinite Integration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Write the general solution of a differential equation and use indefinite integral notation for antiderivatives.<br>-Use basic integration rules to find antiderivatives.<br>-Find a particular solution of a differential equation. | Determine the dy/dx for 2-3 functions. Describe the process using words. | Larsen Text Section: 4.1 Pg: 287-289 |

| 4.2 Area | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Use sigma notation to write and evaluate a sum.<br>-Understand the concept of area.<br>-Approximate the area of a plane region.<br>-Find the area of a plane region using limits. | Find the sum of the numbers from 1 to 100. | Larsen Text Section: 4.2 Pg: 299-301 |

| 4.3 Riemann Sums and Definite Integrals | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Understand the definition of a Riemann sum.<br>-Evaluate a definite integral using limits and geometric formulas.<br>-Evaluate a definite integral using properties of definite integrals.<br>-Approximate a definite integral using the Trapezoidal Rule. | Given a basic definite integral, have students explain what it represents. | Larsen Text Section: 4.3 Pg: 312-315 |

| 4.6 Integration by Substitution | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Use pattern recognition to find a definite integral.<br>-Use a change of variable to find an indefinite integral.<br>-Use the General Power Rule for Integration to find an indefinite integral.<br>-Use a change of variables to evaluate a definite integral.<br>-Evaluate a definite integral involving an even or odd function. | Solve an integral with trigonometric functions. | Larsen Text Section: 4.6 Pg: 343-346 |

| 4.7 Natural Log Function Integration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Use the Log Rule for Integration to integrate a rational function.<br>-Integrate trigonometric functions. | Error analysis problem in Larsen text on pg. 344 #82. | Larsen Text Section: 4.7 Pg: 353-355 |

| 7.2 Integration by Parts | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Find an antiderivative using integration by parts. | Write down the integration by parts formula. Write down at least two things you notice. Talk with a partner about what you notice. | Larsen Text Section: 7.2 Pg: 469-472 |

| 7.3 Trigonometric Integrals | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Solve trigonometric integrals involving powers of sine, cosine, secant and tangent.<br>-Solve trigonometric integrals involving sines and cosines of different angles. | Write down the Pythagorean trigonometric identities. | Larsen Text Section: 7.3 Pg: 479-481 |

| 7.4 Trigonometric Substitutions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Use trigonometric substitution to find an integral.<br>-Use integrals to model and solve real-life applications. | Have students solve the Pythagorean trigonometric identities for sin, cosine and tangent. | Larsen Text Section: 47.4 Pg: 488-490 |

| 7.5 Partial Fractions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| -Understand the concept of partial fraction decomposition.<br>-Use partial fractions decomposition with linear factors to integrate rational functions.<br>-Use partial fraction decomposition with quadratic factors to integrate rational functions. | Ask students: How can we decompose a function? Give students a rational function with 1/basic quadratic. Allow them to factor the denominator and then see if they can write it as the sum of two fractions. | Larsen Text Section: 7.5 Pg: 498-499 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Projects<br>● End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>● Projects<br>● Practice AP Exam Questions<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● ELL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Career Readiness, Life Literacies, and Key Skills<br>● LGBT and Disabilities Law<br>● Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

### Applications of Integration

Students will use integration to find the area between two curves. They will also use rotation and other techniques to find the volume of a variety of 3-D solids as well as of solids with known cross-sectional areas. The First Fundamental Theorem of Calculus will play a major role in the development in understanding of both area and volume, and students will have to rely on a variety of integration techniques to help them progress through the chapter including: substitution, transcendental and trigonometric techniques.

| Essential Questions | Enduring Understandings |
|---|---|
| 1. What is the Fundamental Theorem of Calculus?<br>2. How do you find the area of a region between two curves?<br>3. How can you use integrals to find the volume of a solid?<br>4. How can you use definite integrals to find the arc length of a smooth curve and the area of a surface of revolution? | 1. The Fundamental Theorem of Calculus connects differentiation and integration.<br>2. Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.<br>3. Definite integrals allow us to solve problems involving the accumulation of change in length over an interval. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Definite Integrals closed interval revolution arc length | Fundamental Theorem of Calculus Disc Method Washer Method |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target** *(Specific behavioral objectives that describe the skills students will learn)* | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Evaluate a definite integral using the Fundamental Theorem of Calculus.<br>2. Find the average value of a function over a closed interval.<br>3. Find the area of a region between two curves using integration.<br>4. Describe integration as an accumulation process.<br>5. Find the volume of a solid of revolution using the disc method.<br>6. Find the volume of a solid of revolution using the washer method.<br>7. Find the arc length of a smooth curve. | 1. HSA.SSE.B, HSG.MG.A3<br>2. HSA.SSE.B, HSG.MG.A3<br>3. HSA.SSE.B, HSG.MG.A3<br>4. HSA.SSE.B, HSG.MG.A3<br>5. HSG.GMD.A3<br>6. HSG.GMD.A3<br>7. HSG.GPE.B.7 | 1. 1.D<br>2. 2.D<br>3. 4.C<br>4. 1.E<br>5. 2.D<br>6. 4.E<br>7. 3.D |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| **Section:** 4.4 **Page:** 322<br>**Description:** Calculating the speed of sound | Find the average value of a function over a closed interval. | SCI.HS.PS4-1 |
| **Section:** 6.2 **Page:** 429<br>**Description:** 3-D printing using volume rotated around the y-axis. | Find the volume of a solid of revolution using the washer method. | TECH.8.2.12.D.3 |
| **Section:** 6.4 **Page:** 449<br>**Description:** Calculating the length of a parabolic cable on a suspension bridge. | Find the arc length of a smooth curve. | SCI.HS.LS2-7 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| - Explain the difference between finding area and volume.<br>- Write how to find an area of a region using cross sections. How is this different from rotations? |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 4.4 The Fundamental Theorem of Calculus | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Determine the average value of a function using definite integrals.<br>Interpret the meaning of a definite integral in accumulation problems.<br>Evaluate a definite integral using the Fundamental Theorem of Calculus.<br>Calculate a definite integral using areas and properties of definite integrals. | Determine the area under the curve given an interval. | Larson Text Section 4.4 Pg: 326-328 |

| 6.1 Area of a Region Between Two Curves | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Find the area of a region between two curves using integration.<br>Find the area of a region between intersecting curves using integration.<br>Describe integration as an accumulation process. | Given a picture of a shaded area and the equations bounding it, have students find the area. | Larson Text Section 6.1 Pg: 416-419 |

| 6.2 Volume: The Disk and Washer Method | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Find the volume of a solid of revolution using the disk method.<br>Find the volume of a solid of revolution using the washer method.<br>Find the volume of a solid with known cross sections. | Students can explain the difference between a disk and a washer. How would their difference affect the volume of a rotating object? | Larson Text Section 6.2 Pg: 427-430 |

| 6.4 Arc Length and Surfaces of Revolution | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Find the arc length of a smooth curve.<br>Find the area of a surface of revolution. | Think of an example of when you would need to find the arc length that is not a circle. | Larson Text Section 6.4 Pg: 446-449 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Projects<br>● End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>● Projects<br>● Practice AP Exam Questions<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● ELL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Career Readiness, Life Literacies, and Key Skills<br>● LGBT and Disabilities Law<br>● Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

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## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

### Unit 6: Differential Equations

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth, students build understanding of topics introduced in Algebra II and other courses.

| Essential Questions | Enduring Understandings |
|---|---|
| 1. How do you approximate the particular solution of a differential equation?<br>2. How are differential equations used in application problems, such as the exponential growth and decay models?<br>3. How do you solve separable differential equations?<br>4. How do you solve logistic differential equations? | 1. Solving differential equations allows us to determine functions and develop models. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Exponential Growth and Decay  Differential Equation | Slope Field  Euler's Method  Separable<br>Particular Solution  General Solution  Order |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target** *(Specific behavioral objectives that describe the skills students will learn)* | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Estimate solutions to differential equations.<br>2. Interpret verbal statements of problems as differential equations involving a derivative expression and verify solutions to differential equations.<br>3. Determine general and particular solutions to differential equations.<br>4. Interpret the meaning of a differential equation and the logistic growth model in its context. | 1. HSS.ID.C<br>2. HSF.IF.C.8b<br>3. HSA.APR.D<br>4. HSA.APR.D | 1. 1.E, 2.C, 4.D<br>2. 2.C, 3.G<br>3. 1.E<br>4. 3.G, 3.F |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section:*** 5.1 ***Page:*** 377<br>***Description:*** Temperature - finding the general solution and comparing it to the exact solution. | Estimate solutions to differential equations. | SCI.HS.SS3-5 |
| ***Section:*** 5.2 ***Page:*** 382<br>***Description:*** Using exponential growth model to estimate population growth over time. | Interpret verbal statements of problems as differential equations involving a derivative expression and verify solutions to differential equations. | SCI.HS.SS3.B |
| ***Section:*** 5.3 ***Page:*** 390<br>***Description:*** Modeling advertising awareness using separation of variables. | Determine general and particular solutions to differential equations. | CLKS-FP.6, CLKS-FP.7 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| Explain the difference between a differential and a derivative.<br>Explain how slope fields and numerical methods can be used to solve differential equations without finding an algebraic solution.<br>Describe the difference between a particular solution and a general solution.<br>Explain the difference between growth and decay. |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 5.1 Slope Fields and Euler's Method | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Use initial conditions to find particular solutions of differential equations.<br>Use slope fields to approximate solutions of differential equations.<br>Use Euler's Method to approximate solutions of differential equations. | Students will sketch a line with the following slopes, -3, -1, 0, 1, 5. | Larson Text Section: 5.1 Pg: 375-378 |

| 5.2 Growth and Decay | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Use separation of variables to solve a simple differential equation.<br>Use exponential functions to model growth and decay in applied problems. | Students will sketch the graph of $y = \frac{1}{2}e^{2x}$<br>How can you make 1 change to make the graph represent an exponential decay function? | Larson Text Section: 5.2 Pg: 384-386 |

| 5.3 Separation of Variables | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Recognize and solve differential equations that can be solved by separation of variables.<br>Use differential equations to model and solve applied problems. | Solve the following:<br>$\int_0^1 \frac{24x}{(4x^2+4)^2}$ | Larson Text Section: 5.3 Pg: 393-396 |

| 5.4 The Logistic Equation | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| Solve and analyze logistic differential equations.<br>Use logistic differential equations to model and solve applied problems. | Match the slope field for 3 differential equations. | Larson Text Section: 5.4 Pg: 402-403 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Projects<br>● End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>● Projects<br>● Practice AP Exam Questions<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● ELL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Career Readiness, Life Literacies, and Key Skills<br>● LGBT and Disabilities Law<br>● Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

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## AP CALCULUS BC

## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| Unit Title: Parametric Equations, Polar Coordinates, and Vectors | |
|---|---|
| In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What are parametric equations and how do you find a set of parametric equations to represent a curve?<br>2. How can you find the slope of a tangent line and the arc length of a curve using a set of parametric equations?<br>3. What is the polar coordinate system and the properties of curves written in polar form?<br>4. How do you integrate and find arc length of curves in polar coordinates?<br>5. What are vectors and how do you perform operations using vectors?<br>6. What are vector-valued functions and how do you apply Calculus to them?<br>7. How do you describe velocity and acceleration associated with vector-valued functions? | 1. Derivatives allow us to solve real-world problems involving rates of change.<br>2. Definite integrals allow us to solve problems involving the accumulation of change in length over and interval.<br>3. Recognizing opportunities to apply derivative rules can simplify differentiation<br>4. Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane.<br>5. Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions. |

| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | | | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* | |
|---|---|---|---|---|
| Planar Curve | Smooth | Derivative | Parameter | Parametric Equation |
| Tangent | Area | Arc Length | Polar Coordinates | Polar Graph |
| Vectors | Scalar | Limits / Continuity | Vector-Valued Function | |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| **Unit Learning Targets/Goals/Outcomes** | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Find a set of parametric equations to represent a curve.<br>2. Find the slope of a tangent line to a curve given by para. eqs.<br>3. Understand the polar coordinate system, including graphing.<br>4. Find the area of a region bounded by a polar graph.<br>5. Perform vector operations and interpret the results geometrically.<br>6. Differentiate and integrate vector-valued functions.<br>7. Describe velocity and acceleration with vector-valued functions. | 1. HSA.REI.D<br>2. HSA.SSE.B<br>3. HSF.IF.C<br>4. HSF.IF.C<br>5. HSN.VM.B<br>6. HSN.VM.A<br>7. HSN.VM.A.3 | 1. 1E, 2D<br>2. 1D, 1E<br>3. 2D<br>4. 2D, 3D<br>5. 1D<br>6. 1D, 1E<br>7. 1E |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 9.2* ***Page**: 654*<br>***Description:*** *Projectile motion of a baseball* | Find a set of parametric equations to represent a curve. | SCI.HS-PS2.A |
| ***Section**: 9.5* ***Page**: 679*<br>***Description:*** *Radiation from a transmitting antenna* | Find the area of a region bounded by a polar graph. | SCI.HS.PS4.B |
| ***Section**: 9.6* ***Page**: 687*<br>***Description:*** Tension of a cable supporting a load | Perform vector operations and interpret the results geometrically. | SCI.HS-PS2.A |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |

- Explain the process of sketching a plane curve given by parametric equations. What is meant by the "orientation" of the curve?
- Explain how to find the arc length of a smooth curve given by $x = f(t)$ and $y = g(t)$ on the interval $[a, b]$.
- Describe the differences between the polar and rectangular coordinate systems.
- How are the slopes of tangent lines determined in polar coordinates?
- What are tangent lines at the pole and how are they determined?
- Consider two forces of equal magnitude acting on a point. When the magnitude of the resultant is the sum of the magnitude of the two forces, make a conjecture about the angle between the two forces. What happens when the resultant of the two forces is 0?
- State the definition of continuity of a vector-valued function and give an example of a vector-valued function that is defined, but not continuous, at $t = 2$.
- Compare integration of a vector-valued function with integration of a real-valued function.
- Explain how a particle can be accelerating even though its speed is constant.

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 9.2 - Plane Curves and Parametric Equations | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Sketch the graph of a curve given by a set of parametric equations<br>- Eliminate the parameter in a set of parametric equations.<br>- Find a set of parametric equations to represent a curve | Explain how to write any ordered pair on the unit circle using trig functions and the angle $\theta$ formed by the positive x-axis and the point on the circle. | Larson Text Section: 9.2 Pg: 652-654 |

| 9.4 - Polar Coordinates and Polar Graphs | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Understand the polar coordinate system.<br>- Rewrite rectangular coordinates and equations in polar form and vice versa.<br>- Sketch the graph of an equation given in polar form.<br>- Find the slope of a tangent line to a polar graph.<br>- Identify several types of special polar graphs. | Evaluate 3-4 trigonometric values of angle measurements | Larson Text Section: 9.4 Pg: 670 - 672 |

| 9.6 - Vectors in the Plane | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Write the component form of a vector.<br>- Perform vector operations and interpret the results geometrically. | - Define the terms 'magnitude' and 'direction' in your own words in relation to mathematics.<br>- Try and recall how to add two vectors geometrically using an xy coordinate plane. | Larson Text Section: 9.6 Pg: 686 - 688 |

| 9.3 - Parametric Equations and Calculus | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Find the slope of a tangent line to a curve given by a set of parametric equations.<br>- Find the arc length of a curve given by a set of parametric equations. | - Eliminate the parameter and write the corresponding equation in rectangular coordinates.<br>- Create a set of parametric equations using trig functions that describe the path of a particle moving on the unit circle from (1,0) to (0,1) in a counterclockwise motion. | Larson Text Section: 9.3 Pg: 659-62 |

| 9.5 - Area and Arc Length in Polar Coordinates | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - FInd the area of a region bounded by a polar graph.<br>- Find the points of intersection of two polar graphs.<br>- Find the arc length of a polar graph. | - Converting polar coordinates to rectangular and vice versa (3-4 examples)<br>- If integration in rectangular coordinates uses sums of rectangles, what shape do you think we will be adding up if our representation is in polar form? Justify your conjecture. | Larson Text Section: 9.5 Pg: 687 - 680 |

| 9.7 - Vector-Valued Functions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Analyze and sketch a plane curve given by a vector-valued function.<br>- Extend the concepts of limits and continuity to vector-valued functions.<br>- Differentiate a vector-valued function.<br>- Integrate a vector-valued function. | - Find the following vector magnitudes (3 examples)<br>- What are the three criteria for a function to be continuous at a point in the real-valued function case? | Larson Text Section: 9.7 Pg: 695 - 697 |

| 9.8 - Velocity and Acceleration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Describe the velocity and acceleration associated with a vectored-value function. | - Find derivatives for the following vector-valued functions. | Larson Text Section: 9.8 Pg: 702-703 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

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<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.</td>
<td>The following assessments require students to utilize various strands of mathematics.<ul><li>Projects</li><li>Practice AP Exam Questions</li><li>Homework</li><li>Classwork</li></ul></td>
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</table>

| State Mandates and Resources |
|---|
| • New Jersey Student Learning Standards<br>• Career Readiness, Life Literacies, and Key Skills<br>• LGBT and Disabilities Law<br>• Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC

## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Infinite Series** | |
|---|---|
| In this unit, students need to understand that a sum of infinitely many terms may converge to a finite value. They can develop intuition by exploring graphs, tables, and symbolic expressions for series that converge and diverge and for Taylor polynomials. Students should build connections to past learning, such as how evaluating improper integrals relates to the integral test or how using limiting cases of power series to represent continuous functions relates to differentiation and integration. Students who rely solely on memorizing a long list of tests and procedures typically find little success achieving a lasting conceptual understanding of series. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. How do you write terms of a sequence and determine whether the sequence converges or diverges?<br>2. What is a convergent infinite series?<br>3. How do you use the Integral Test or properties of special series (p and Harmonic) to determine whether or not an infinite series converges or diverges?<br>4. How do you use comparison tests to determine the convergence of infinite series?<br>5. What is an alternating series and how do you determine its convergence?<br>6. What are the root and ratio tests to determine convergence of series?<br>7. How do you find polynomial approximations (such as Taylor and Maclaurin polynomials) of elementary functions?<br>8. What is a power series and how do you find the radius and intervals of convergence of them? | 1. Applying limits may allow us to determine the finite sum of infinitely many terms.<br>2. Power series allow us to represent associated functions on an appropriate interval |

| | |
|---|---|
| 9. How do you construct power series using operations?<br>10. How do you find Taylor or Maclaurin series for a function? | |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Sequence, Series, Limit<br>Geometric Series, Harmonic Series, Polynomial<br>Ratio, Root, Derivative | Convergence, Divergence, Monotonic<br>Bounded, Partial Sums, Alternating Series<br>Power Series, Radius of Convergence<br>Interval of Convergence |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target**<br>***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Write the terms of a sequence and decide whether a series converges or diverges. | 1. HSF.IF.A.3 | 1. 3D |
| 2. Understand the definition of a convergent infinite series. | 2. HSA.SSE.B4 | 2. 3D |
| 3. Use the Integral Test to determine whether an infinite series converges or diverges. | 3. HSA.REI.A | 3. 3B, 3D |
| 4. Use comparison tests to determine the convergence or divergence of a series. | 4. HSA.REI.D | 4. 3D |
| 5. Use the Alternating Series Test to determine the convergence of a series. | 5. HSA.REI.D | 5. 1E, 3D |
| 6. Use the Ratio or Root Tests to determine whether a series converges or diverges. | 6. HSA.REI.D | 6. 3D |
| 7. Find polynomial approximations (Taylor and Maclaurin) of elementary functions. | 7. HSA.APR.C | 7. 2C, 3D |
| 8. Find the radius/interval of convergence of a power series. | 8. HSA.REI.A | 8. 1F, 2C |
| 9. Find a geometric power series that represents a function. | 9. HSA.APR.C | 9. 3D |
| 10. Find Taylor or Maclaurin series for different elementary functions. | 10. HSA.APR.C | 10. 2C |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 8.1* ***Page**: 542*<br>***Description:** Compound interest from a sequence perspective* | Write the terms of a sequence and decide whether a series converges or diverges. | PFL.9.1.12.D.1 |
| ***Section**: 8.2* ***Page**: 552*<br>***Description:** Multiplier Effect of tourism revenue in a resort city* | Understand the definition of a convergent infinite series. | CAEP.9.2.12.C |
| ***Section**: 8.10* ***Page**: 624*<br>***Description:** Angular projectile motion* | Find Taylor or Maclaurin series for different elementary functions. | SCI.HS-PS2.A |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| - Let $\{a_n\}$ be an increasing sequence such that $2 \leq a_n \leq 4$. Explain why $\{a_n\}$ has a limit. What can you conclude about the limit?<br>- You delete a finite number of terms from a divergent series. Will the new series still diverge? Explain your reasoning.<br>- You add a finite number of terms to a convergent series. WIll the new series still converge? Explain your reasoning.<br>- Define a $p$-series and state the requirements for its convergence.<br>- Can you use the Limit Comparison Test to determine the limit of a series? Explain why or why not.<br>- In your own words, describe the difference between absolute and conditional convergence of an alternating series.<br>- What can you conclude about the convergence or divergence of $\Sigma a_n$ using the Ratio Test when $a_n$ is a rational function of $n$? Explain.<br>- An elementary function is approximated by a polynomial. In your own words, describe what is meant by saying that the polynomial is *expanded about c* or *centered at c*.<br>- Compare the radius of convergence and the interval of convergence of a power series.<br>- Describe three ways to find the Maclaurin Series for $cos^2 x$. Show that each method produces the same first three terms. |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 8.1 - Sequences | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Write the terms of a sequence.<br>- Determine whether a sequence converges or diverges.<br>- Write a formula for the *n*th term of a sequence.<br>- Use properties of monotonic sequences and bounded sequences. | Anything Fibonacci! | Larson Text Section: 8.1 Pg: 542 - 544 |

| 8.2 - Series and Convergence | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Understand the definition of a convergent infinite series.<br>- Use properties of infinite geometric series.<br>- Use the *n*th-Term Test for Divergence of an infinite series. | Use recursive or explicit formulas to write out the first 5 terms of given sequences. | Larson Text Section: 8.2 Pg: 552 - 554 |

| 8.3 - The Integral Test and *p*-Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Use the Integral Test to determine whether an infinite series converges or diverges.<br>- Use properties of *p*-series and harmonic series. | Determine the first 5 partial sums of a set of series.<br>Determine convergence or divergence of given series. | Larson Text Section: 8.3 Pg: 559 - 561 |

| 8.4 - Comparisons of Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Use the Direct Comparison test to determine whether a series converges or diverges.<br>- Use the Limit Comparison Test to determine whether a series converges or diverges. | Error analysis of the use of the Integral Test to determine convergence for 2 given series.<br><br>State the conditions on p-series convergence. | Larson Text Section: 8.4 Pg: 566 - 568 |

| 8.5 - Alternating Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Use the Alternating Series Test to determine whether an infinite series converges.<br>- Use the Alternating Series Remainder to approximate the sum of an alt. series.<br>- Classify a convergent series as absolutely or conditionally convergent.<br>- Rearrange an infinite series to obtain a different sum | Use comparison tests to determine the convergence or divergence of given series. | Larson Text Section: 8.5 Pg: 575 - 576 |

| 8.6 - The Ratio and Root Tests | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Use the Ratio Test to determine whether a series converges or diverges.<br>- Use the Root Test to determine whether a series converges or diverges.<br>- Review all convergence/divergence tests. | List all convergence tests for series that we have learned thus far. Which is your favorite and why? | Larson Text Section: 8.6 Pg: 583 - 585 |

| 8.7 - Taylor Polynomials and Approximations | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Find polynomial approximations of elementary functions and compare them with elementary functions.<br>- Find Taylor and Maclaurin polynomial approximations of elementary functions.<br>- Use the remainder of a Taylor polynomial. | Find equations of tangent lines for given functions through given points.<br><br>Open question. Can you think of any other way to approximate a function with other curves? | Larson Text Section: 8.7 Pg: 594 - 596 |

| 8.8 - Power Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Understand the definition of a power series.<br>- Find the radius and interval of convergence of a power series.<br>- Differentiate and integrate a power series. | Determine the Maclaurin polynomials of degree 3 or 4 for a set of elementary functions | Larson Text Section: 8.8 Pg: 604 - 606 |

| 8.9 - Representation of Functions by Power Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Find a geometric power series that represents a function.<br>- Construct a power series using series operations. | Find the function representation of a set of geometric series with known common ratios and starting terms. | Larson Text Section: 8.9 Pg: 612 - 613 |

| 8.10 - Taylor and Maclaurin Series | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| - Find a Taylor or Maclaurin series for a function.<br>- Find a binomial series.<br>- Use a basic list of Taylor Series to find other Taylor Series. | Use desmos.com to graph the partial sums (Maclaurin polynomial) of the exponential function. What do you notice about the approximation as you add more and more terms? | Larson Text Section: 8.9 Pg: 623 - 625 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

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| State Mandates and Resources |
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AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: AP Test Review** | |
|---|---|
| This unit frameworks the AP Calculus BC content students must know and skills they must master in order to earn transferable, long-term understandings of calculus. The unit reflects a commitment to the previous units where students will elevate their understanding through AP test style questioning, approaches, and breakdowns of test questions. Students will build their test taking skills through copious MCQ and FRQ question styling. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. How do you dissect and apply appropriate calculus techniques to FRQ AP Test questions?<br>2. How do you dissect and apply appropriate calculus techniques to MCQ AP Test questions? | 1. Reasoning with definitions, theorems and properties can be used to justify claims about FRQ style questions.<br>2. Reasoning with definitions, theorems and properties can be used to justify claims about MCQ style questions. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Limit<br>2. Derivative<br>3. Integral<br>4. Differential<br>5. Volume | 1. FRQ<br>2. MCQ |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target** ***(Specific behavioral objectives that describe the skills students will learn)*** | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| • Identify, calculate, and analyze various FRQ and MCQ problems | • ALL High School Mathematics Standards | • ALL |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section****: AP Practice Exam* ***Page****: 714*<br>***Description:*** Farming - Feed Tank Volume | Identify, calculate, and analyze various FRQ and MCQ problems | HS-ESS2-4 |
| ***Section****: AP Practice Exam* ***Page****: 715*<br>***Description:*** Manufacturing to optimize gas tanks | Identify, calculate, and analyze various FRQ and MCQ problems | HS-ESS2-4 |
| ***Section****: AP Practice Exam* ***Page****: 720*<br>***Description:*** Particle Motion | Identify, calculate, and analyze various FRQ and MCQ problems | HS-PS2-2 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively |

| | |
|---|---|
| • Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| **Unit Resources:** | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| **Students will write:** |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| Write the appropriate calculus concept for a real world problem<br>Write justifications for their reasoning on FRQ style problems<br>Short, concise self error analyses about their mock results<br>Definitions of key terms and theorems from the entire course |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| AP Test Review | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Identify, calculate, and analyze various FRQ and MCQ problems | Describe how the three main components of calculus (Limits, Derivatives, and Integrals) are connected. | AP Central FRQ & MCQ |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>• Diagnostic Pre-Test<br>• Chapter Tests<br>• Projects<br>• End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>• Projects<br>• Practice AP Exam Questions<br>• Homework<br>• Classwork |

**List of Accommodations and Modifications**

- Special Education
- 504 Students
- At Risk Students
- ELL
- Gifted and Talented

## State Mandates and Resources

- New Jersey Student Learning Standards
- Career Readiness, Life Literacies, and Key Skills
- LGBT and Disabilities Law
- Asian and Pacific Islander

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
| 1.A Identify the question to be answered or problem to be solved *(not assessed)*. | 2.A Identify common underlying structures in problems involving different contextual situations. | 3.A Apply technology to develop claims and conjectures *(not assessed)*. | 4.A Use precise mathematical language. |
| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
| 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*). | 2.C Identify a re-expression of mathematical information presented in a given representation. | 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied. | 4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$). |
| 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems. | 2.D Identify how mathematical characteristics or properties of functions are related in different representations. | 3.D Apply an appropriate mathematical definition, theorem, or test. | 4.D Use appropriate graphing techniques. |
| 1.E Apply appropriate mathematical rules or procedures, with and without technology. | 2.E Describe the relationships among different representations of functions and their derivatives. | 3.E Provide reasons or rationales for solutions and conclusions. | 4.E Apply appropriate rounding procedures. |
| 1.F Explain how an approximated value relates to the actual value. | | 3.F Explain the meaning of mathematical solutions in context. | |
| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Additional Topics** | |
|---|---|
| This unit is to serve as an additional topics unit to prepare students in their post-secondary mathematical careers. It is essential for discovering and developing important groundwork for multivariate and differential calculus courses. Students will solve net change and shell method problems algebraically and conceptually. They will extend their knowledge from all prior coursework to build both a solid and intuitive understanding of these topics. Strong emphasis will be on communicating both orally and written what the application of their answers mean. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What is the Net Change Theorem?<br>2. How do you determine the volume of a solid using the Shell Method | 1. Reasoning with theorems of calculus to justify values of the rate of change function<br>2. Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Washer<br>2. Disc<br>3. Axis of Rotation<br>4. Cylinder<br>5. Integral | 1. Shell |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target** *(Specific behavioral objectives that describe the skills students will learn)* | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Understand and use the Net Change Theorem<br>2. Find the volume of a solid of revolution using the Shell Method | 1. HSA.SEE.B<br>2. HSG.GMD.A3 | 1. 1D, 2D<br>2. 2D, 4E |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 4.5* ***Page**: 333*<br>***Description:*** Velocity and Initial Position | Understand and use the Net Change Theorem | HS-PS2-2 |
| ***Section**: 6.3* ***Page**: 439*<br>***Description:*** Oblateness of Saturn | Find the volume of a solid of revolution using the Shell Method | HS-ESS3-2 |
| ***Section**: 6.3* ***Page**: 438*<br>***Description:*** Machine Part for Installation | Find the volume of a solid of revolution using the Shell Method | HS-ESS2-4 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
|---|---|
| **21st-Century Skills** | **Mathematical Practices** |
| • Critical thinking<br>• Creativity<br>• Collaboration<br>• Communication<br>• Information literacy<br>• Technology literacy<br>• Media literacy<br>• Flexibility<br>• Leadership<br>• Initiative<br>• Productivity<br>• Social skills | • Make sense of problems and persevere in solving them<br>• Reason abstractly and quantitatively<br>• Construct viable arguments and critique the reasoning of others<br>• Model with mathematics<br>• Use appropriate tools strategically<br>• Attend to precision<br>• Look for and make use of structure<br>• Look for and express regularity in repeated reasoning |

| Unit Resources: | | |
|---|---|---|
| **Textbook** | **Online Resources** | **Calculators** |
| Calculus for AP, 2nd Ed, Larson/Battaglia | https://apcentral.collegeboard.org<br>https://sso.cengage.com/cb<br>WebAssign<br>CalcChat.com<br>CalcView.com | TI-84 Plus<br>TI-89<br>Desmos.com |

| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| How can integrals be simplified to be able to use techniques we know<br>Solutions to complex rational functions through integration<br>Explain the difference between Disc, Washer, and Shell Methods<br>Compare Net Change Theorem to the Fundamental Theorem of Calculus<br>Algebraic and conceptual proofs relating the relationships between NCT and FTC |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| 4.5 - The Net Change Theorem | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Understand and use the Net Change Theorem | Describe, in words, the fundamental theorem of calculus | Larson Text Section: 4.5 Pg: 333 - 335 |

| Section number and Title: | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • How do you determine the volume of a solid using the Shell Method | Determine the volume of a shape rotated on a given Axis of Rotation (Disc or Washer) | Larson Text Section: 6.3 Pg: 436 - 439 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
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<td>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<ul><li>Diagnostic Pre-Test</li><li>Chapter Tests</li><li>Projects</li><li>End-Of –Course Assessment</li></ul></td>
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| State Mandates and Resources |
|---|
| • New Jersey Student Learning Standards<br>• Career Readiness, Life Literacies, and Key Skills<br>• LGBT and Disabilities Law<br>• Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

| Practice 1 | Practice 2 | Practice 3 | Practice 4 |
|---|---|---|---|
| ***Implementing Mathematical Processes*** 1<br>Determine expressions and values using mathematical procedures and rules. | ***Connecting Representations*** 2<br>Translate mathematical information from a single representation or across multiple representations. | ***Justification*** 3<br>Justify reasoning and solutions. | ***Communication and Notation*** 4<br>Use correct notation, language, and mathematical conventions to communicate results or solutions. |

## SKILLS

| | | | |
|---|---|---|---|
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| 1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*. | 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. | 3.B Identify an appropriate mathematical definition, theorem, or test to apply. | 4.B Use appropriate units of measure. |
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| | | 3.G Confirm that solutions are accurate and appropriate. | |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS BC
## 034000

**Updated:** July 2023

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Marginal Analysis Project** | |
|---|---|
| In this unit, students will complete a real-world project that reflects the applications of differential calculus in business. Students will have to research key definitions for Marginal Revenue, Cost, and Profit and how they are calculated through the use of calculus. Students will then need to create and provide a full solution to a real-world problem addressing at least one of the key terms. A verbal presentation will conclude the topic with the support of a Powerpoint summarizing their findings and solutions. | |
| **Essential Questions** | **Enduring Understandings** |
| 1. What is Marginal Revenue?<br>2. What is Marginal Profit?<br>3. What is Marginal Cost?<br>4. How do you use calculus to determine Marginal Revenue, Profit, and/or Cost? | 1. Marginal analysis is utilized as a decision making tool to maximize potential profits for businesses. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Derivative<br>2. Function<br>3. Revenue<br>4. Cost<br>5. Profit | 1. Marginal |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| Unit Learning Targets/Goals/Outcomes | | |
|---|---|---|
| **Learning Target** *(Specific behavioral objectives that describe the skills students will learn)* | **NJSLS:** | **Mathematical Practices (see AP table the end of the document):** |
| 1. Define, Calculate, and Analyze Marginal Profits, Cost, and Revenues for Real-World Problems | 1. HSA.CED.A | 1. 1E, 3D, 4B |

| Interdisciplinary Connections | | |
|---|---|---|
| **Real-world problem-solving examples** | **Learning Target** | **NJSLS** |
| ***Section**: 1.2* ***Page**: 74* ***Description:*** Cost analysis functions for paddle board company | Define, Calculate, and Analyze Marginal Profits, Cost, and Revenues for Real-World Problems | CLKS 9.1 |

| The following 21st-century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects: | |
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| Students will write: |
|---|
| *Students will define and compare/contrast given terms. Students will describe and write about a diagram using mathematical language. Students will relate real-world situations using geometry terminology. Students will also prove postulates and theorems* |
| Definitions of Marginal Revenue, Cost, and Profit<br>Create a virtual presentation of key terms and solutions to real world problems<br>Justifications as to why corporations would want to utilize the interpretations of solutions<br>Utilize proper differentiation notation within solutions |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section number and Title: | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Define, Calculate, and Analyze Marginal Profits, Cost, and Revenues for Real-World Problem | Derive the given function | Larson Text Section: 2.5 Pg: 180-182 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

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AP CALCULUS AB AND BC

# Mathematical Practices

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